AD-A088 580 INSTITUTE FOR DEFENSE ANALYSES ARLINGTON VA SCIFNCE A--ETC F/G 16/4.2
A COMPARISON OF THE CHARACTERISTICS OF THREE SAMPLING SCHEMES F-- TC(U)
MAR 80 V J BERINATI, J H HENRY MDA903-79-C-0018
UNCLASSIFIED IDA-P-1478 IDA/HQ-80-22061 NL

1 OF 1
AD A
088580

END
DATE
FILMED
10-80
DTIC

AD-E500 197

Copy 18 of 75 copies

(12) LEVEL III

IDA PAPER P-1478

A COMPARISON OF THE CHARACTERISTICS OF THREE SAMPLING SCHEMES FOR THE VERIFICATION INSPECTION OF CERTAIN MX ICBM SYSTEMS

AD A088580

V. J. Berinati
J. H. Henry

March 1980

DTIC ELECTE SEP 2 1980 S D B

Prepared for

Office of the Under Secretary of Defense for Research and Engineering

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

IDA

INSTITUTE FOR DEFENSE ANALYSES
SCIENCE AND TECHNOLOGY DIVISION

DDC FILE COPY

80 8 25 181

IDA Log No. HQ 80-220

The work reported in this document was conducted under contract MDA 903 79 C 0018 for the Department of Defense. The publication of this IDA Paper does not indicate endorsement by the Department of Defense, nor should the contents be construed as reflecting the official position of that agency.

Approved for public release; distribution unlimited

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO. AD-A088 580	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A Comparison of the Characteristics of Three Sampling Schemes for the Verification Inspection of Certain MX ICBM Systems		5. TYPE OF REPORT & PERIOD COVERED Final
		6. PERFORMING ORG. REPORT NUMBER IDA Paper P-1478
7. AUTHOR(s) V. J. Berinati, J. H. Henry		8. CONTRACT OR GRANT NUMBER(s) MDA 903 79 C 0018
9. PERFORMING ORGANIZATION NAME AND ADDRESS Institute for Defense Analyses, 400 Army-Navy Drive, Arlington, VA 22202		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Task Order T-9-045
11. CONTROLLING OFFICE NAME AND ADDRESS Assistant to the Secretary of Defense (Atomic Energy). Attn: Director. SALT/Arms Control Support Group, OATSD (AE)		12. REPORT DATE March 1980
		13. NUMBER OF PAGES 56
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) DoD-IDA Management Office, 400 Army-Navy Drive, Arlington, Virginia 22202		15. SECURITY CLASS (of this report) Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE NA

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

ICBM	Sampling	Vertical Shelter
Ballistic Missile	MX	Probability of Detection
Arms Control	Multiple Protective Structure	Horizontal Loading Dock
Verification	Horizontal Dock	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Three schemes, proposed by others, for sampling certain MX ICBM systems for purposes of arms limitation verification inspection are evaluated. The principal criterion is the probability of detecting excess missiles as a function of the number of shelters to be sampled. Other criteria are the geographical distribution of the shelters, the number of occupied and empty shelters disclosed, and the need of a master list of missile locations.

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

IDA PAPER P-1478

A COMPARISON OF THE CHARACTERISTICS OF THREE SAMPLING SCHEMES FOR THE VERIFICATION INSPECTION OF CERTAIN MX ICBM SYSTEMS

V. J. Berinati
J. H. Henry

March 1980

INSTITUTE FOR DEFENSE ANALYSES
SCIENCE AND TECHNOLOGY DIVISION
400 Army-Navy Drive, Arlington, Virginia 22202

Contract MDA 903 79 C 0018
Task T-9-045

ABSTRACT

Three schemes, proposed by others, for sampling certain MX ICBM systems for purposes of arms limitation verification inspection are evaluated. The principal criterion is the probability of detecting excess missiles as a function of the number of shelters to be sampled. Other criteria are the geographical distribution of the shelters, the number of occupied and empty shelters disclosed, and the need of a master list of missile locations.

DTIC
S ELECTE D
SEP 2 1980
B

ACCESSION for
NTIS White Section
DDC Buff Section
UNANNOUNCED
JUSTIFICATION
BY
DISTRIBUTION/AVAILABILITY CODES
Dist. AVAIL. and/or SPECIAL
A

FOREWORD

This study was performed under Task T-9-045, SALT Studies, for the Office of the Assistant to the Secretary of Defense, Atomic Energy (SALT/Arms Control Support Group), OUSDRE.

CONTENTS

1. INTRODUCTION

In the consideration of means to verify the US MX intercontinental ballistic missile system, there is a possibility that some type of periodic inspection of the deployed system may be required to provide assurance to the inspector that significant numbers of illegal missiles have not been introduced. Schemes for sampling the force have been proposed, aimed at providing reasonable probabilities of detection (P_d) of an illegal missile without requiring the inspection of an excessive number of shelters. Such sampling is desirable for all current MX deployment modes, including the Horizontal Dash Multiple Protective Structure (MPS) now under development, the Horizontal Loading Dock MPS, and the Vertical Shelter MPS.

Other factors are also important in comparing sampling schemes; the geographical distribution of the shelters to be inspected, the number of occupied and empty shelters disclosed in the inspection, and the possibility of a need for the existence of a master list of deployed missile locations. The characteristics of three proposed inspection schemes are determined and compared in this study. The schemes are: TRW Method II, Schultis 2, and Cooper's Sample and Search.

Inspection sampling schemes that have been proposed have, for illustrative purposes, used deployment models with various numbers of shelters, legal missiles, and illegal missiles. In this study, the three schemes are examined for a single model: 4,000 shelters, 200 legal missiles, and from 20 to 200 (= M) illegal missiles for cheating fractions (CF = 1 + M/200) from 1.1 to 2.0. Probabilities of detection are presented for the

numbers of shelters inspected (N) from 20 to 100; in Cooper's scheme, these are expected numbers of shelters inspected.

Certain cheating strategies available to the deployer are investigated. These strategies pertain only to the deployer's response to a stated sampling technique, e.g., lying against Schultis 2 or, against Cooper's method, selecting an optimum number of groups in which to place illegal missiles. Other forms of cheating, such as thwarting the random selection of shelters to be inspected, are not investigated.

2. TRW METHOD II

2.1 DESCRIPTION

The numbered statements following immediately constitute the verbatim description presented in Ref. 1. Following the numbered statements there is a supplemental description provided by the authors of this paper.

1. Deployer, inspector, and neutral certify complete (1-4000) inner ball set, numbered in accordance with agreed shelter designations.
2. Neutral draws all balls individually.
3. Neutral places wrapped, numbered two-piece cover over inner ball.
4. Deployer sensor reads, tabulates (securely) ball number--cover number correlation (and carefully self-checks).
5. Balls placed in hopper and mixed. (Shortcuts available if perfect memories ruled out.)
6. Neutral draws and unwraps all 4000 balls.
7. Deployer removes and immediately destroys 200 balls and declares no remaining missiles in shelters designated by remaining inner balls.
8. Random drawing to determine N declared empty shelters for inspection.
9. Inner balls exposed for N empty shelters.
10. Remaining balls (alleged empty shelters) immediately destroyed.

Each numbered ball represents a shelter. It has a differently numbered cover and the deployer knows the correlation

between ball number and cover number. Each cover is then wrapped so the neutral can draw them honestly; after the first drawing the wrappers are removed. Because the deployer knows the correlation between the number on each cover and the number on each ball, he can then remove the covered balls representing the shelters containing the 200 legal missiles; all of the remaining covered balls represent 3800 empty shelters. In a second drawing from the 3800 covered balls, the agreed-upon number (N) of shelters is selected for inspection and the covers removed from the N balls to identify the shelters to be inspected.

2.2 DEPLOYER'S CHEATING STRATEGY

None known.

2.3 CHARACTERISTICS

The probabilities of detection are shown in Fig. 1; details of the analysis are presented in Appendix A. The shelters to be inspected would be geographically widely distributed and, if the deployer used operational grouping of shelters in 20 shelters per group, it would be expected that the shelters to be inspected would be in N groups (if $N < 200$). The locations of no occupied shelters are disclosed in a legal deployment; the locations of N unoccupied shelters are disclosed. The deployer's representative must possess a master list of missile locations.

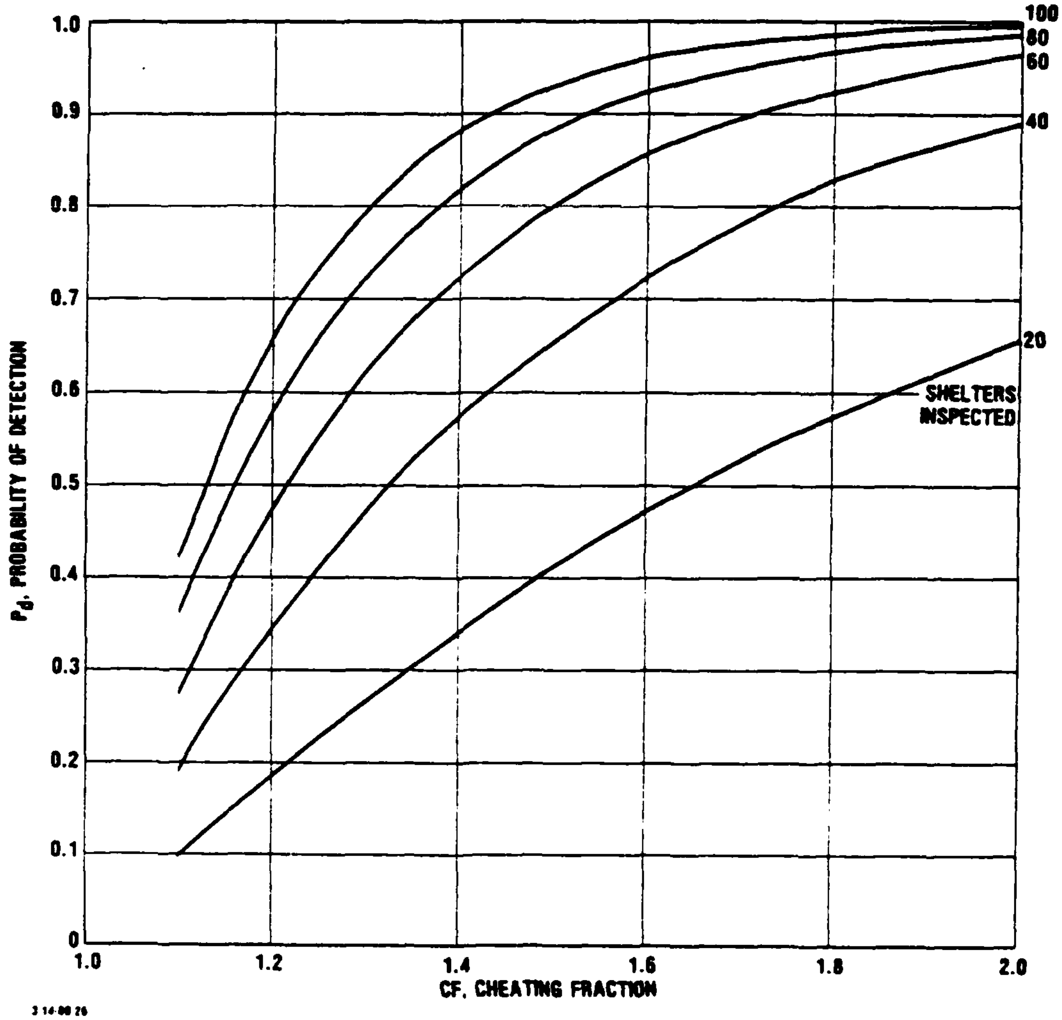

FIGURE 1. TRW Method II - Probability of Detection

3. SCHULTIS 2

3.1 DESCRIPTION (REF. 2)

1. A neutral provides a set of balls (4000) which are each numbered. Each number is the serial number of a shelter.
2. The deployer provides the neutral with a map showing the relative location of each shelter.
3. The balls are placed in a basket in view of the deployer and the inspector.
4. Twenty balls are drawn from the basket; these 20 shelters constitute a set.
5. The neutral proceeds to select more balls to constitute a total of 200 sets.
6. The shelter numbers which constitute each set are revealed to the deployer.
7. The deployer reports for each set the exact number of missiles which are in that set of 20 shelters.
8. As the sets are declared by the deployer, the inspector places all 0-missiles sets (each set consisting of 20 balls) in one basket, all 1-missile sets in a second basket, etc.
9. The inspector selects from this total of 200 sets the specific sets (1 to 5 in this study) to be inspected, selecting each set from the 0-missile, or 1-missile, etc., lots.

3.2 DEPLOYER'S CHEATING STRATEGY

There are two possible cheating strategies, differing in what the cheating deployer would declare as the distribution of the numbers of sets containing various numbers of missiles. In the second column of Table 1 is shown the distribution that would exist if the deployer had no illegal missiles, i.e., there would be 71 0-missile sets, 76 1-missile sets, etc; we term this the legal distribution.

TABLE 1. STATISTICAL DISTRIBUTION OF SETS CONTAINING VARIOUS NUMBERS OF MISSILES

1	2	3	4
	0 Illegal Missiles	120 Illegal Missiles	
Number of Missiles In One Set	Expected Number Of Sets	Expected Number Of Sets	MCPD* Declaration
0	71	37	52
1	76	67	96
2	38	54	52
3	12	28	0
4	3	10	0
Total	200	196 (~200)	200

* MCPD = Minimum Common Probability of Detection

If variations from this expected legal distribution (Column 2 of Table 1) are unlikely, the deployer would be forced to declare the legal distribution; this case is treated in Section 3.3.1. If variations are unlikely, the deployer may be able to declare another distribution and it seems there is an optimum lie for the deployer; this case, the minimum common probability of detection (MCPD) distribution, is treated in Section 3.3.2.

An investigation has been made of the likelihood of the occurrence of the number of 0-missile sets in a legal deployment (shown in Table 1 as 71 0-missile sets; the exact hypergeometric solution is 71.54 sets). The results are shown in Fig. 2 and the analysis is described in Section A.2.2.3 of the Appendix. In Section 3.3.2 there are presented the probabilities that the numbers of 0-missile sets which are required for the MCPD declarations would occur.

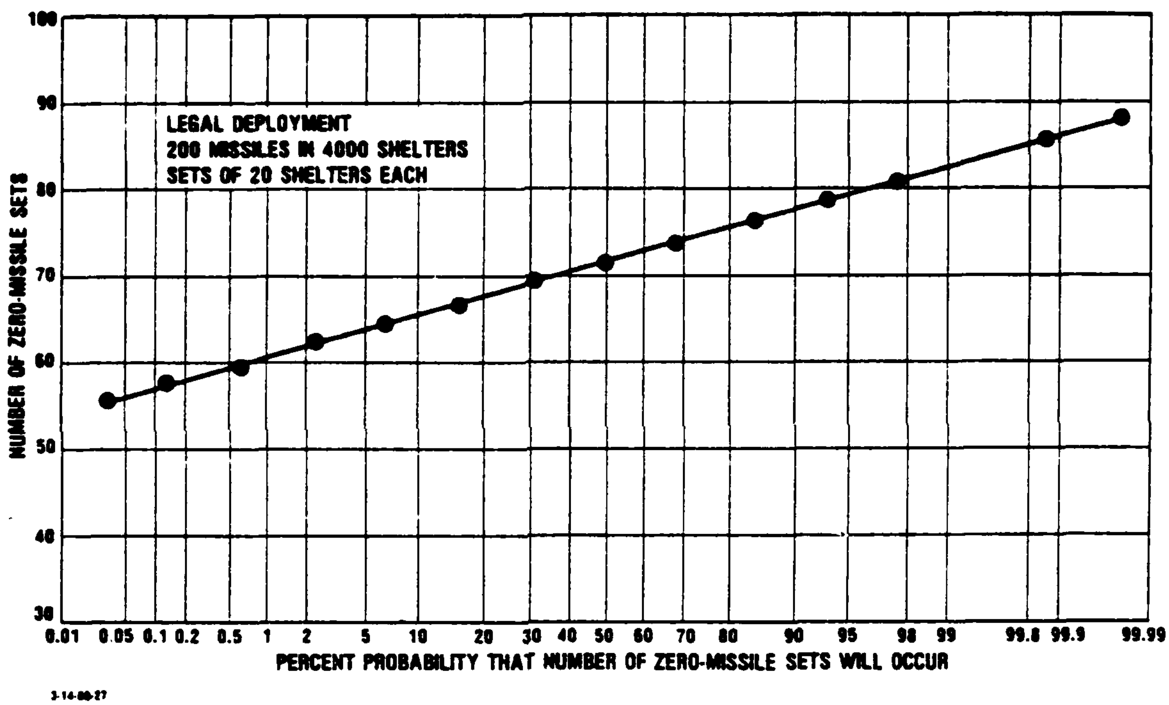

FIGURE 2. Probability of 0-missile sets occurring

3.3 CHARACTERISTICS

3.3.1 Legal Distribution

The probabilities of detection are presented in Fig. 3. The inspector always chooses 0-missile sets; his probability of detection is lower in 1-missile sets and is zero in 2-missile, 3-missile, and 4-missile sets. The probabilities of

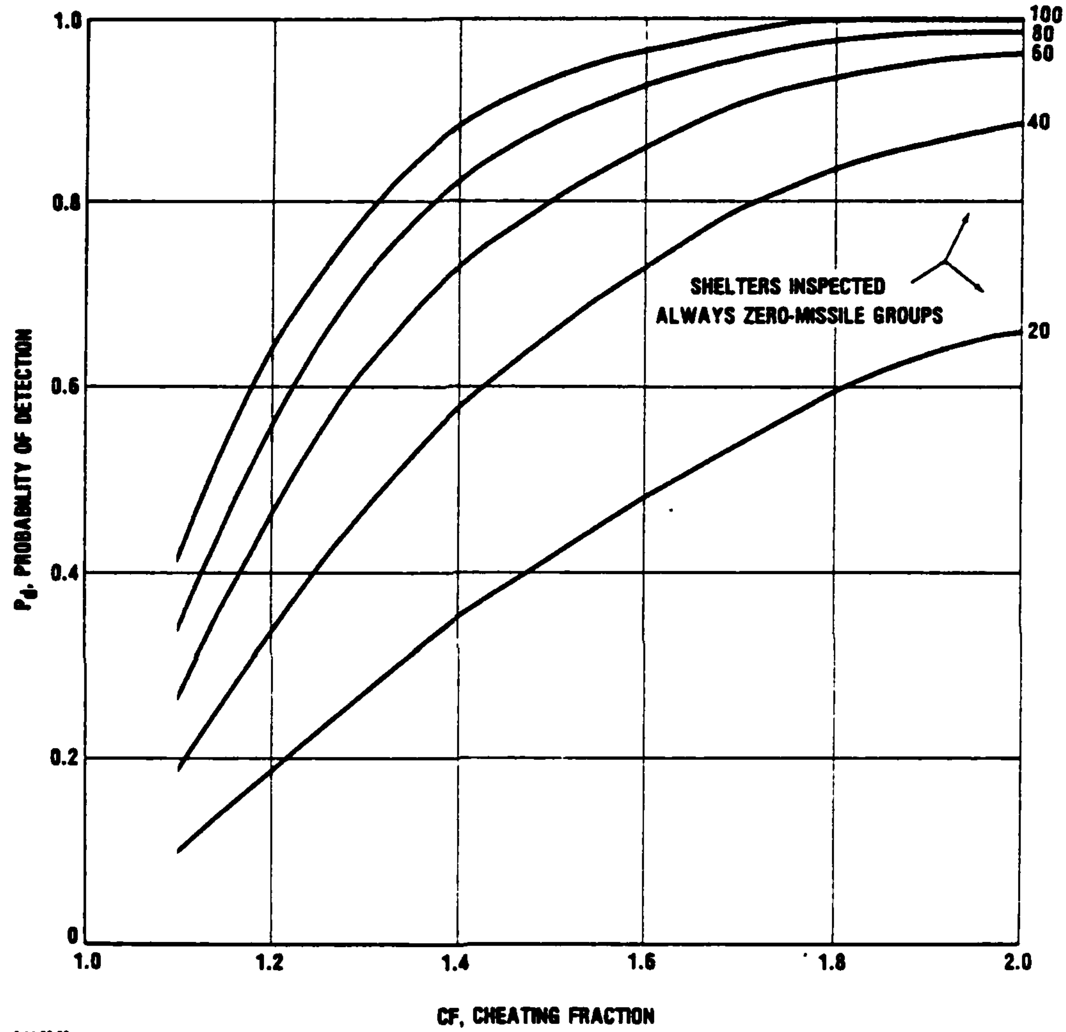

FIGURE 3. Schultis 2 probability of detection. Deployer declares legal distribution.

detection appear to be the same as those for TRW Method II (compare with Fig. 1). As shown in Section A.2.2.2 of the Appendix, the solutions are identical for inspection of 20 shelters. The solutions are very close for more than 20 shelters and may, in fact, also be mathematically identical if a different form of analysis had been used.

The shelters to be inspected would be geographically widely distributed and, if the deployer used operational grouping of shelters in 20 shelters per group, it would be expected that the shelters to be inspected would be in N groups (if $N < 200$). For 20, 40, 60, 80, and 100 shelters inspected (=N), the expected number of occupied shelters of which the locations would be disclosed will be 1, 2, 3, 4, and 5, respectively (= 0.05 N) and the expected number of unoccupied shelters of which the locations would be disclosed will be 19, 38, 57, 76, and 95, respectively (= 0.95 N). The deployer's representative would create a list of missile locations only among the N shelters to be inspected. (The representative must have access to a system which would identify how many missiles are occupying each set of 20 shelters. Such a system could give a user the ability to locate all missiles in the deployment with enough time and enough interrogations. It may be possible to design the system to prevent such abuse.)

3.3.2 MCPD Distribution

One particular technique for the deployer's false declarations of distributions has been used, i.e., to first reduce the number of 4-missile sets (placing them among the less-than-4-missile sets while always retaining a total of 200 sets and 200 missiles), then reducing the number of 3-missile sets, and finally reducing the number of 2-missile sets. Some other techniques were investigated but yielded higher probabilities of detection; still, it is not certain at this time that a better technique does not exist.

The inspector is driven to inspect either 0-missile or 1-missile sets to obtain a positive probability of detection. Using as an example the case for M = 120 illegal missiles (cheating fraction of 1.6) the probabilities of detection are 0.479 and 0.118, respectively, for the legal distribution of 71 sets of 0-missile and 76 sets of 1-missile. As the declared number of 0-missile sets is reduced and, correspondingly, the declared number of 1-missile sets is increased, the probabilities of detection in the 0-missile sets decrease and those in the 1-missile sets increase. Finally, at 52 0-missile sets $P_d = 0.288$ and at the corresponding 96 1-missile sets $P_d = 0.302$, approximately equal (at this point 52 2-missile sets would also be declared but they are all in fact 2-missile sets so the $P_d = 0$). This is the "minimum common probability of detection" (MCPD) declaration; the inspector must choose either 0-missile or 1-missile sets and he has the same probability of detection in either. The situation is depicted in Columns 3 and 4 of Table 1.

The probabilities of detection are presented in Fig. 4. For these probabilities the deployer must declare 67 0-missile sets if 20 illegal missiles were deployed, 63 if 40 illegal missiles were deployed, and 59 if 60 illegal missiles were deployed. From Fig. 2 the possibilities that these numbers of 0-missile sets would occur are 24, 4, and 0.4 percent, respectively. It appears there is little opportunity for the deployer to utilize the MCPD declaration unless the number of illegal missiles is small. Other characteristics of the MCPD declaration for a legal deployment are the same as those for the legal distribution of Section 3.3.1.

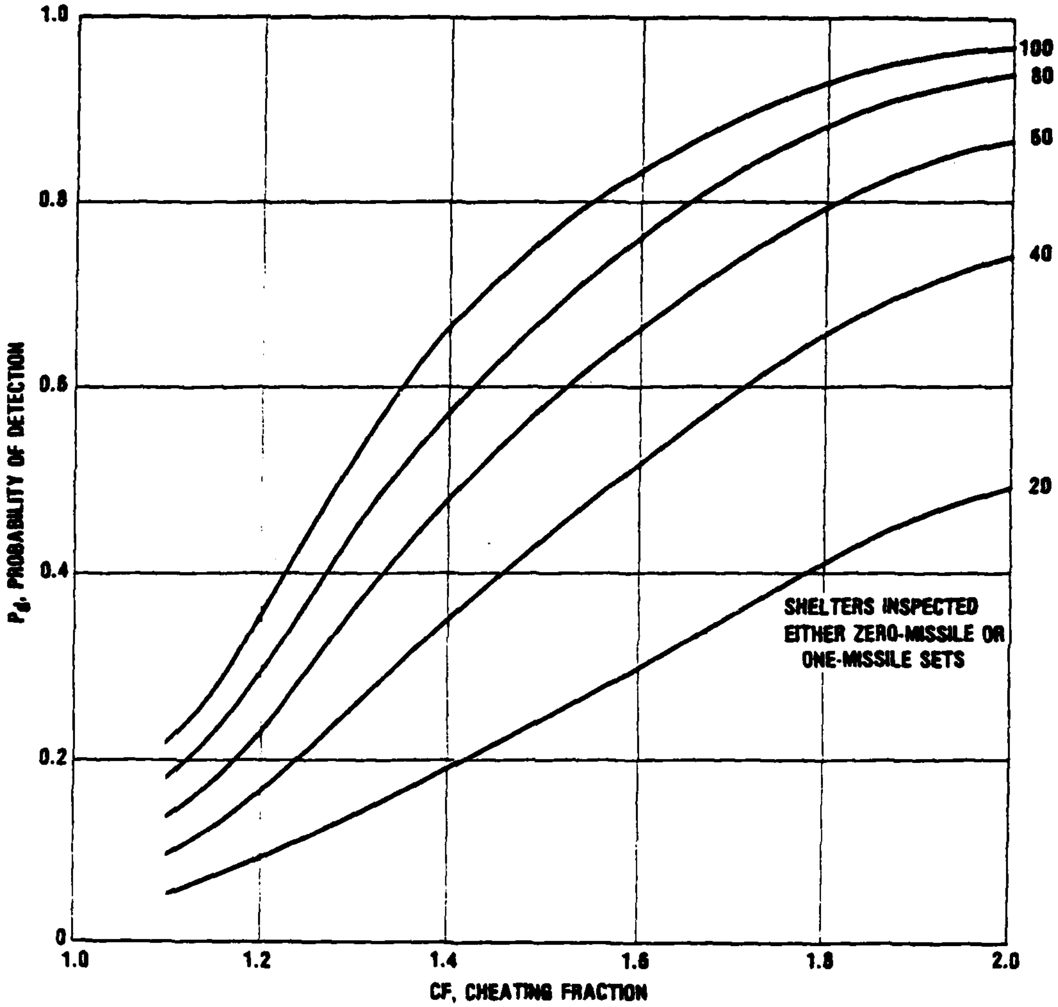

FIGURE 4. Schultis 2 probability of detection. Deployer declares minimum-common P_d distribution.

4. COOPER'S SAMPLE AND SEARCH

4.1 DESCRIPTION (Ref. 3)

For this paper the deployment is assumed to consist of 200 (G) groups of 20 shelters each with one legal missile per group.

1. The inspector samples N1 groups by inspecting one shelter in each group. Any group in which a missile is found is a missile-bearing group. (In this analysis N1 was examined for 5, 10, 15, 20, 40, 60, 80, and 100.)
2. The inspector then is permitted to inspect one or more (A) of the additional shelters in each missile-bearing group; this is called the search phase. (In this analysis A was examined for 4, 9, 10, 11, 12, 13, 14, and 19.)

4.2 DEPLOYER'S CHEATING STRATEGY

If an illegal number of missiles M were deployed it is assumed that they would be distributed equally among a number of illegal groups (G1). This opportunity is permitted the deployer in the analysis under the assumption that he knows the inspection plan, i.e., N1 and A. The probabilities of detection are determined with the deployer using the best cheating strategy (BCS). The BCS is different for different values of N1 and A, and for M except when N1 = G and A = 19, in which case the BCS expressed as M/G1 is the same for all M.

PRECEDING PAGE BLANK-NOT FILMED

4.3 RESULTS

4.3.1 Sample All Groups, Search Missile-Bearing Groups Fully ($N1 = G$, $A = 19$)

The probabilities of detection are presented in Fig. 5 for the BCS which is 5 illegal missiles per group (M/G1) for all numbers of illegal missiles (M). The expected number of shelters inspected $N = 390 + 0.95M$, is also shown in Fig. 5.

The number of shelters inspected is probably larger than that of interest, so this scheme is not investigated further; neither is the scheme for sampling all groups ($N1 = G$) and searching missile-bearing groups partially ($A < 19$) for the same reason.

4.3.2 Sample Part of Groups, Search Missile-Bearing Groups Fully ($N1 < G$, $A = 19$)

The probabilities of detection are presented in Fig. 6 for the BCS which varies from 10 to 16 illegal missiles per group (primarily dependent on the number of illegal missiles, M). As noted in Fig. 6, the number of groups sampled (N1) varies from 7 to 50; this figure is driven primarily by the number of shelters inspected (N), i.e., when only 20 shelters are to be inspected the number of groups sampled must be small since any missile-bearing group would be searched fully (incurring an additional 19 shelters inspected per missile-bearing group).

The expected number of shelters to be inspected $N = 1.95N1 + 0.00475N1\ M$. N1 is also the number of groups to be inspected and, for a legal deployment, $N1 = 0.513N$, so that when $N = 20$ ten groups would be inspected, and when $N = 100$ fifty-one groups would be inspected. The inspected groups would therefore be widely distributed, but the inspected shelters would be in a more limited number of groups as compared with TRW Method II and Schultis 2. In a legal deployment, the locations of 0.025N occupied shelters would be expected to be disclosed (1 for $N = 20$

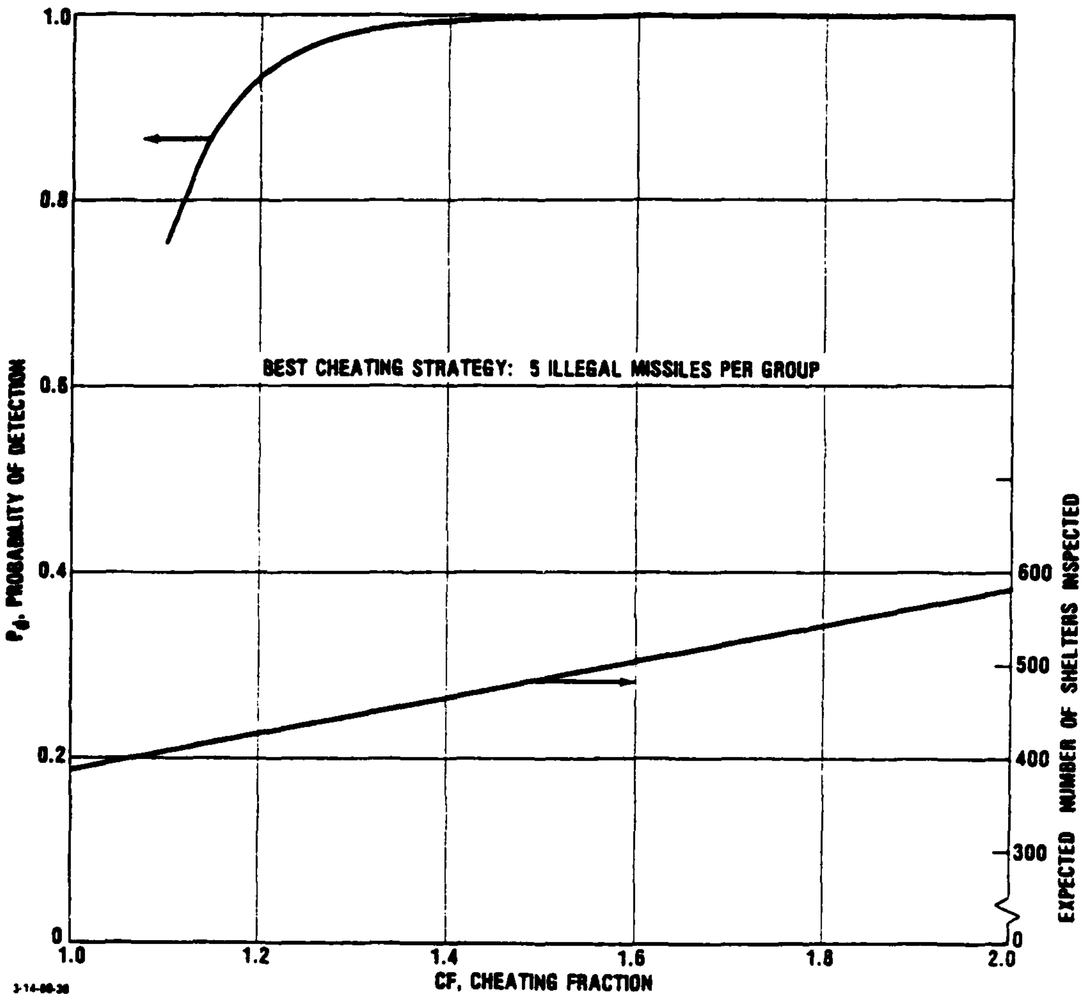

FIGURE 5. Cooper's sample and search probability of detection. Sample all groups (N1 = G), search missile-bearing groups fully (A = 19).

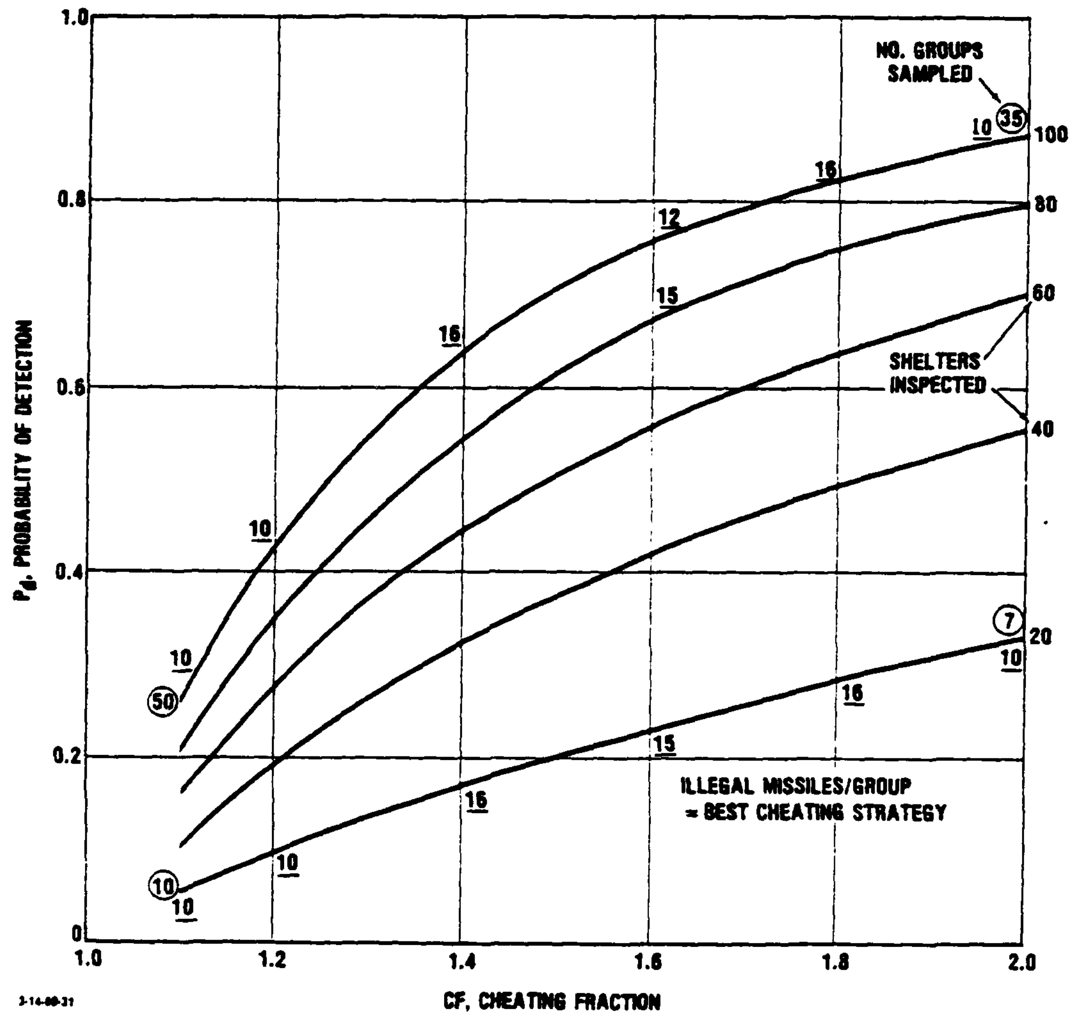

FIGURE 6. Cooper's sample and search probability of detection. Sample part of groups. Search missile-bearing groups fully.

and 3 for N = 100), and 0.975N unoccupied shelters (19 for N = 20 and 97 for N = 100). Access to information on missile locations is not required.

4.3.3 Sample Part of Groups, Search Missile-Bearing Groups Partially ($N1 < G$, $A < 19$)

Cases for A = 4, 9, 10, 11, 12, 13, and 14 additional shelters to be searched in each missile-bearing group were examined and the case for A = 11 found to yield the highest probability of detection for all M and N. The probabilities of detection are shown in Fig. 7 for A = 11 and they are superior to those shown in Fig. 6 where A = 19. The BCS is identified in Fig. 7; for N > 79 it is the smallest M/G1 available (= 1) and for N < 70 it is the largest M/G1 available.

The expected number of shelters to be inspected $N = N1 + 0.05A\ N1 + 0.00025A\ N1\ M$ or, when A = 11, $N = 1.55N1 + 0.00275N1\ M$. N1 is also the number of groups to be inspected and, for a legal deployment $N1 = 0.645N$, so that when N = 20 thirteen groups would be inspected, and when N = 100 sixty-four groups would be inspected. The inspected groups would therefore be widely distributed, but the inspected shelters would be in a more limited number of groups as compared with TRW Method II and Schultis 2 (but more widely distributed than with Cooper, A = 19). In a legal deployment, the locations of 0.032N occupied shelters would be expected to be disclosed (1 for N = 20 and 3 for N = 100), and 0.968N unoccupied shelters (19 for N = 20 and 97 for N = 100). Access to information on missile locations is not required.

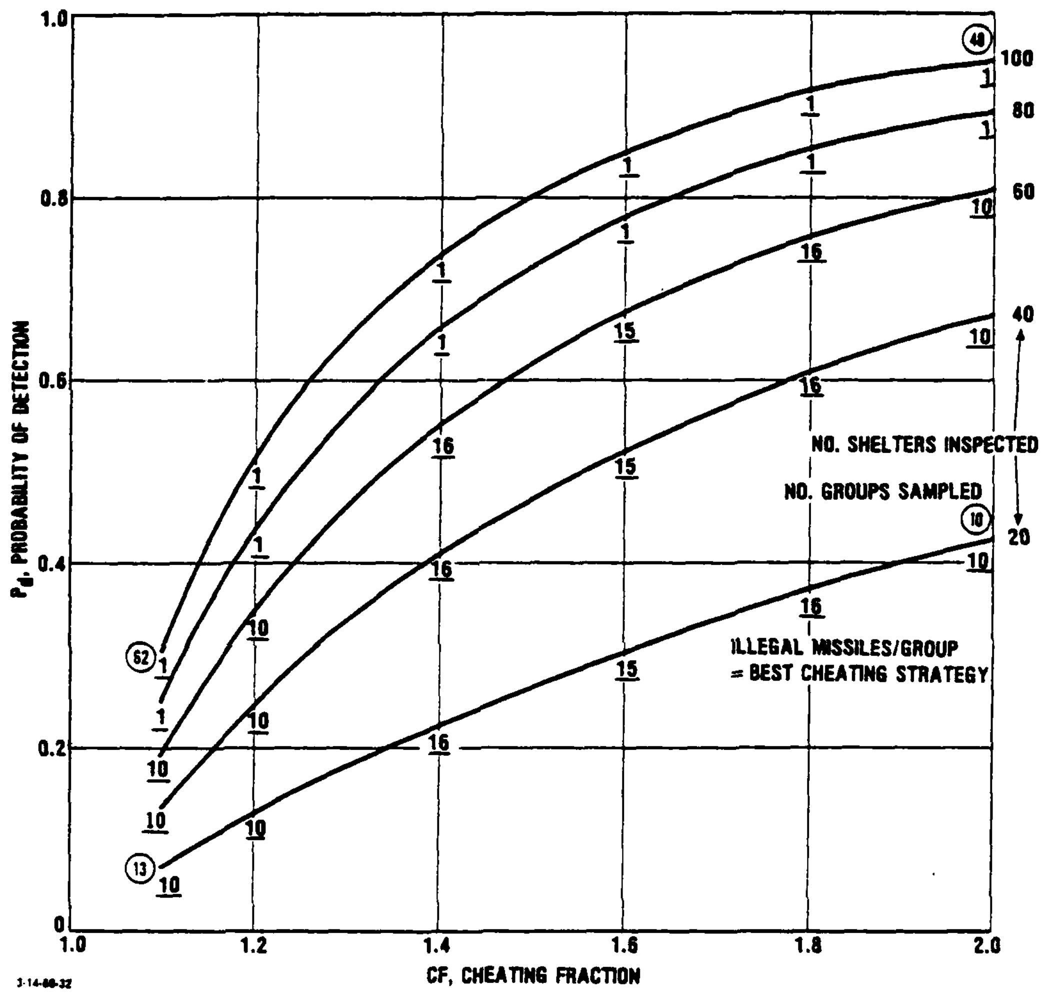

FIGURE 7. Cooper's sample and search probability of detection. Sample part of groups. Search missile-bearing groups partially. Optimum search: 11 additional shelters per missile-bearing group.

5. COMPARISON

In Fig. 8 the probabilities of detection are compared for the three methods for 20 and 100 shelters inspected. Recall that the analysis indicates that the deployer may be able to use the Schultis 2 Minimum Common Probability of Detection (MCPD) declaration only for small illegal deployments--20 illegal missiles or fewer. TRW Method II and Schultis 2 legal (identical) are better than Cooper for 20 to 100 shelters inspected and 0 to 200 illegal missiles. The probabilities of detection with 20 shelters inspected are likely lower than is desired, although the values are surprisingly high (TRW II and Schultis legal) for such a small number of inspections when the cheating is high. At low levels of cheating, the probabilities of detection, even for 100 shelters inspected, are liable to be unacceptable.

One hundred shelter inspections, or perhaps even a higher number, is probably a tolerable figure in operation. Table 2 compares the methods for 100 shelters inspected, listing all of the criteria. Comparisons are shown for 20 and 200 illegal missiles. A deployer with 200 illegal missiles may well be ready to initiate a conflict; from exchange analyses, 200 additional missiles represents a potent force. Twenty illegal missiles is the other end of the scale; the acceptability of 10 percent illegal missiles may depend upon the frequency of the inspections. In Table 2 some characteristics are shown for no illegal missiles; these are of interest when one considers the perils and disruption generated by the legal deployer being inspected routinely.

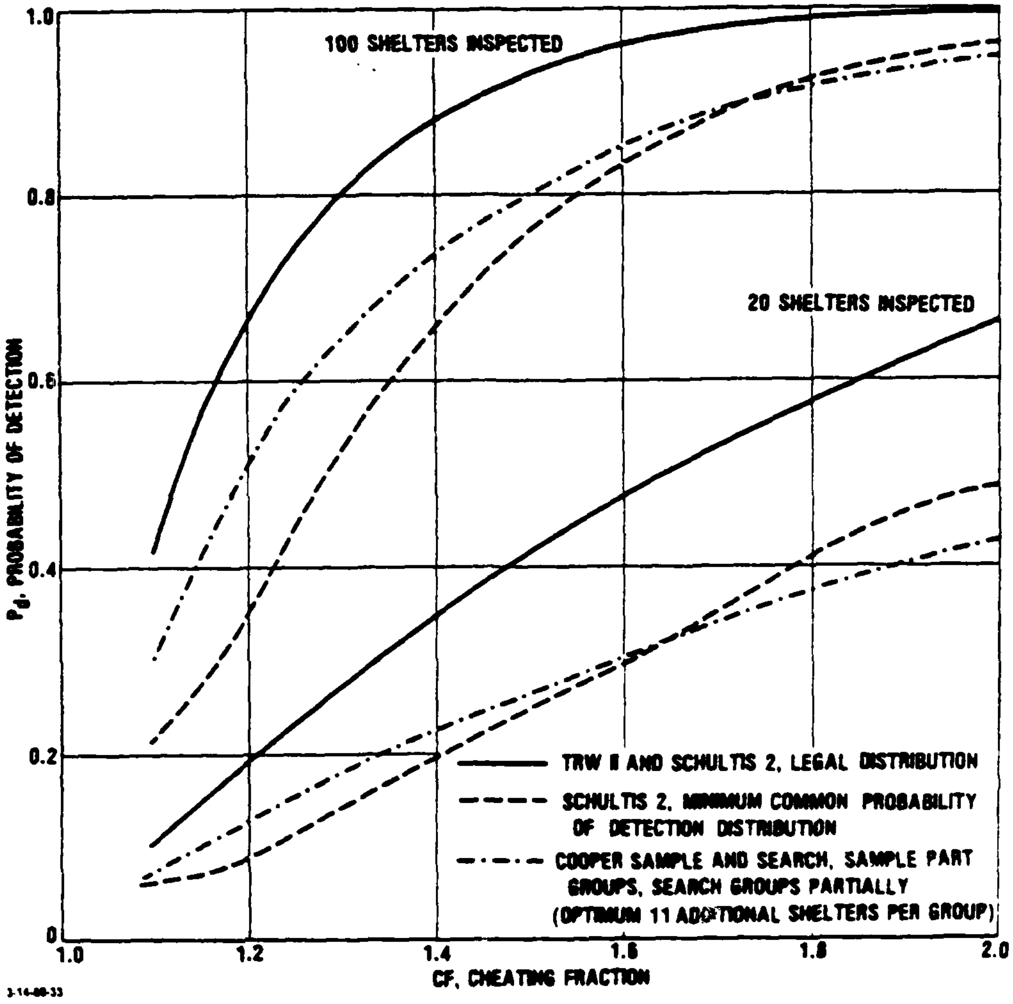

FIGURE 8. Comparison of probabilities of detection. 4000 shelters, 200 legal missiles.

All methods are subject to a wide geographic dispersal in the locations of the shelters to be inspected. If one considers the deployer having organized in 20-shelter groups, one would expect these inspected shelters to be in more groups for the TRW and Schultis methods than for Cooper's.

TABLE 2. COMPARISON OF METHODS, 100 SHELTERS INSPECTED

4000 Shelters, 200 Legal Missiles

	0 Illegal Missiles			20 Illegal Missiles				200 Illegal Missiles			
	TRW II	Schultis Legal	Cooper A=11	TRW II	Schultis Legal	Schultis MCPD	Cooper A=11	TRW II	Schultis Legal	Schultis MCPD	Cooper A=11
Probability of Detection				0.42	0.42?	0.22	0.31	1.00	1.00	0.96?	0.95
Number of Groups Inspected	100	100	64	100	100	100	62	100	100	100	48
Best Cheating Strategy, Illegal Missiles per Group							1				1
Listing of Missile Locations Required	Yes	Access	No	Yes	Access	Access	No	Yes	Access	Access	No
Geographic Dist. of Shelters Inspected	Wide	Wide	Wide								
No. of Occupied Shelters, Loc. Disclosed	0	5	3								

The Schultis probabilities of detection have been marked with a ? for the legal declaration at 20 illegal missiles and for the MCPD declaration at 200 illegal missiles. As explained earlier, with 20 illegal missiles the deployer has a chance of getting away with the necessary lie for the MCPD declaration, so one should probably consider it as feasible and ignore the figure for the legal declaration. With 200 illegal missiles the deployer seems to have little chance of getting away with the necessary lie for the MCPD declaration, so one should ignore the figure for it (in any event, it is little different from the figure for the legal declaration).

With 20 illegal missiles the probabilities of detection are all somewhat low, with the TRW method best and the Schultis MCPD method poorest. With 200 illegal missiles the probabilities of detection are all very high and, for all practical purposes, identical.

TRW's method requires the existence of a list of all missile locations; Cooper's method requires none. Schultis' method requires the existence of a system which would identify how many missiles are occupying each set of 20 shelters. Such a system could give a user the ability to locate all missiles in the deployment with enough time and enough interrogations. It may be possible to design the system to prevent such abuse.

TRW's method does not disclose the locations of any occupied shelters; Cooper's method discloses 3 and Schultis' method discloses 5. Some believe that disclosure of the location of an occupied shelter would give the inspector's country the opportunity to validate a clandestine missile-detection system. Dr. Schultis has suggested a procedure which would avoid the disclosure of the location of any occupied shelters for the Schultis 2 or Cooper methods. In brief, when a set (or group) of shelters has been identified for inspection and the deployer has declared the number of missiles in that set, the deployer

would be permitted to visit that set with a number of missile-transporters equal to the number of missiles declared, and remove the missiles before inspection. The use of such a procedure would make all three methods equal, i.e., no disclosure of the location of any occupied shelter, for this criterion.

No single method appears superior by all measures. With 200 illegal missiles one may prefer Cooper's method, accepting the slightly lower probability of detection in consideration of the smaller number of groups inspected and the lack of a need for any list of missile locations. With 20 illegal missiles (and some consider inspection as aimed at detecting this level of illegality) Cooper's probability of detection is significantly lower than that for TRW Method II and both probabilities are already relatively low; Cooper's advantage of not needing a list of missile locations may not be worth the lower probability of detection.

REFERENCES

1. J.M. Gorman, TRW Defense and Space Systems Group Briefing 7904-U-27175-TRW, "Shelter Sampling Techniques," April 1979.

2. W.J. Schultis, Institute for Defense Analyses Memorandum, "A Scheme for Selection of the Specific Holes to be Inspected in MPS," 9 March 1979.

3. H.F. Cooper, R and D Associates Briefing, "MAP Verification Options," June 8, 1978.

PRECEDING PAGE BLANK-NOT FILMED

APPENDIX

DETAILS OF THE SAMPLING ANALYSES

APPENDIX

DETAILS OF THE SAMPLING ANALYSES

A.1 TRW METHOD II

The inspector is offered 3800 shelters (= S) in which, while it is alleged there are no illegal missiles (= M), there may be in this analysis M = 20, 40, ... 180, or 200 illegal missiles. The inspector may select some number of shelters (= N) for verification; in this analysis N = 20, 40, 60, 80, or 100 shelters.

This is a hypergeometric problem because when the inspector selects a number of shelters for inspection, say N = 20, his probability of selecting a missile-bearing shelter is different on his first selection when there are, say M = 120 missile-bearing shelters in a lot of S = 3800 shelters, from his probability when, on his twentieth selection there are M = 120 missile-bearing shelters in a lot of 3781 shelters.

The probability, P10, of finding no missile-bearing shelter is (Ref. A-1, Chapter II, Equation 6.1):

$$P10 = \frac{\binom{M}{K'} \binom{S - M}{N - K'}}{\binom{S}{N}} \qquad \text{(A-1)}$$

where K' = the number of missile-bearing shelters to be found in the sample of size N, = 0.

The probability, P11, of finding one or more missile-bearing shelters, which is therefore the probability of detection of a cheat, is:

$$P11 = 1 - P10 \quad . \tag{A-2}$$

$$P11 = 1 - \frac{\binom{M}{0}\binom{S - M}{N - 0}}{\binom{S}{N}} = 1 - \frac{1\binom{S - M}{N}}{\binom{S}{N}} \quad . \tag{A-3}$$

$$P11 = 1 - \frac{(S - M)!}{(S - M - N)!} \frac{(S - N)!}{S!} \quad . \tag{A-4}$$

The probabilities of detection, P11, are plotted in Fig. 1 for N = 20, 40, 60, 80, and 100 shelters, M = 20 to 200 missiles (equivalent to cheating fractions of 1.1 to 2.0, since there are 200 legal missiles in this study so that CF = 1 + M/200), and S = 4000 - 200 = 3800 shelters (the 200 shelters are removed before the inspector selects shelters for inspection).

A.2 SCHULTIS 2

A.2.1 Expected Distribution of a Legal Deployment

The neutral is offered 4000 shelters (=S) in which, while it is alleged there are no illegal missiles (=M), there may be in this analysis M = 20, 40, ... 180, or 200 illegal missiles. The neutral draws successively 200 sets of 20 shelters each and the deployer declares how many missile-bearing shelters there are in each set.

This is also a hypergeometric problem. The neutral will finally have drawn some numbers of 0-missile, 1-missile, 2-missile, etc. sets (as declared so by the deployer) numbering 200 sets in total and, whether the deployment is legal or not, accounting for 200 missiles. The probability, P12k, of drawing a k-missile set of 20 shelters from a lot of s shelters containing m missiles (s may be considered equal to S = 4000 or some

smaller number; m may correspondingly be equal to M = 200 or some smaller number; see following discussion) is:

$$P12k = \frac{\binom{m}{k}\binom{s - m}{20 - k}}{\binom{s}{20}} \qquad \text{(A-5)}$$

As it develops, for all practical purposes, k runs from 0 to no more than 6; i.e., there is only a very small probability that there will be any 7-or-greater-missile sets when the ratio, missiles/shelters, is 1/10 to 1/20.

To perform the analysis properly these probabilities should be determined for 200 successive drawings of 20-shelter sets with the first draw from the legal deployment of s = 4000 shelters with 200 missiles, the second draw from s = 3980 shelters with 199 missiles, etc., and the average probability ($\overline{P12}k$) calculated for each k-missile set over the 200 draws. This has been done but is not used in the analysis; this "proper" expected number (rounded to an integer) of k-missile sets (=200 $\overline{P12}k$) is shown in the second column of Table A-1 for the legal deployment of 200 missiles.

TABLE A-1. EXPECTED NUMBER OF k-MISSILE SETS, LEGAL DEPLOYMENT

Set	Proper Number of Sets	Analysis Number of Sets, G_o
0-missile	70	71
1-missile	77	76
2-missile	38	38
3-missile	12	12
4-missile	2	3
5-missile	0	0
6-missile	0	0
Number of Sets	199	200
Number of Missiles	197	200

In the analysis, the probabilities of the first draw of the "proper" technique are used to determine the expected numbers of k-missile sets (=200 P12k, s = S = 4000). The resulting distribution for the legal deployment, G_o, (m = 200) is shown in the third column of Table A-1; in this legal distribution and some of the other distributions with illegal missiles it was necessary to adjust the number of sets by one for some k-missile sets in order to produce the totals of 200 sets and 200 + M missiles.

A.2.2 Declaration of Legal Distribution

The deployer may cheat by choosing some declaration of the distribution of the numbers of k-missile sets so as to lessen the probability that his cheating would be detected. The most extreme of these false declarations, and therefore the most beneficial to the deployer, we term the minimum common probability of detection (MCPD) and it is treated in Section A.2.3.

At the other end of the scale of possible declarations by the deployer is the declaration of the legal distribution; this case is treated in this section. In Section A.2.2.3 an estimate is made of the confidence limit of the legal distribution which sheds light on the issue of whether the deployer could reasonably declare a distribution of sets other than the legal distribution.

A.2.2.1 Analysis. The probabilities (again, of the first draw of the "proper" technique) are used to determine the number of k-missile sets (=200 P12k, s = S = 4000) for the illegal deployments of m = 220, 240, ... 400 total missiles, corresponding to M = 20, 40, ... 200, respectively. The results, rounded to integers and adjusted to produce 200 total sets and m total missiles, are shown in Table A-2.

TABLE A-2. EXPECTED NUMBER OF k-MISSILE SETS, VARIOUS DEPLOYMENTS

	Number of Illegal Missiles, M										
Set	0	20	40	60	80	10C	120	140	160	180	200
0-missile	71	64	58	51	46	41	37	34	29	27	24
1-missile	76	75	74	73	71	68	67	63	60	57	54
2-missile	38	42	45	49	51	54	54	55	58	57	57
3-missile	12	15	17	20	23	26	28	31	34	36	38
4-missile	3	4	5	6	7	9	10	12	14	16	19
5-missile	0	0	1	1	2	2	3	4	4	5	6
6-missile	0	0	0	0	0	0	1	1	1	1	2
Number of Sets	200	200	200	200	200	200	200	200	200	199	200
Number of Missiles	200	220	240	260	280	300	320	340	360	374	400

The data of Table A-2 show that the inspector should not select a set for inspection if it contains two or more missiles. If illegal missiles are deployed, the number of high-k sets available is always in excess of the number for a legal distribution; the deployer's declaration for high-k sets will always be verified upon inspection.

G_M is the expected number of k-missile sets (i.e., the figures in the main portion of Table A-2) and is assumed to be the actual number of k-missile sets when M illegal missiles are deployed. If the deployer declares the distribution of the legal deployment (G_o), the probability of detection, $P13_o$, among the 0-missile sets is:

$$P13_o = 1 - \left(\frac{G_M}{G_o}\right)^{N/20} , \qquad \text{(A-6)}$$

where G_M and G_o are selected from the 0-missile line in Table A-2 and N = number of shelters inspected.

The probability of detection, $P13_1$, among 1-missile sets is taken as:

$$P13_1 = 1 - \left(\frac{G_M}{G_o}\right)^{N/20} , \qquad \text{(A-7)}$$

where G_M and G_o are selected from the 1-missile line in Table A-2.

The probability of detection for a given M and N is always greater for the 0-missile sets; the inspector only inspects 0-missile sets. The results are shown in Fig. 3.

Different results would be obtained if, instead of using the rounded and adjusted Gs, one used the expected hypergeometric probabilities or the "proper" analysis. For $M = 120$ and $N = 20$ the probabilities of detection would be:

Analysis	0.4789
Hypergeometric	0.4746
"Proper"	0.4778

A.2.2.2 Similarity to TRW Method II. A comparison of Figs. 1 and 3 shows that the probabilities of detection are virtually the same.

In the analysis of Schultis 2, from Eq. (A-5) with $k = 0$:

$$G_o = 200 \frac{\binom{200}{0}\binom{4000 - 200}{20}}{\binom{4000}{20}} \qquad \text{(A-8)}$$

$$G_M = 200 \frac{\binom{200 + M}{0}\binom{4000 - 200 - M}{20}}{\binom{4000}{20}} \qquad \text{(A-9)}$$

which, from Eq. (A-6), yields:

$$P13_o = 1 - \left[\frac{\binom{200 + M}{0}\binom{3800 - M}{20}}{\binom{200}{0}\binom{3800}{20}}\right]^{N/20} \qquad \text{(A-10)}$$

$$P13_o = 1 - \left[\frac{(3800 - M)!\ 3780!}{(3780 - M)!\ 3800!}\right]^{N/20} \qquad \text{(A-11)}$$

From Eq. (A-4), TRW Method II with S = 3800 shelters shows the probability of detection as:

$$P11 = 1 - \frac{(3800 - M)!\ \ (3800 - N)!}{(3800 - M - N)!\ \ \ 3800!} \quad . \qquad \text{(A-12)}$$

When N = 20, Eq. (A-11), Schultis 2 becomes:

$$P13_o = 1 - \frac{(3800 - M)!\ 3780!}{(3780 - M)!\ 3800!} \quad . \qquad \text{(A-13)}$$

When N = 20, Eq. (A-12), TRW Method II becomes:

$$P11 = 1 - \frac{(3800 - M)!\ 3780!}{(3780 - M)!\ 3800!} \quad . \qquad \text{(A-14)}$$

For this case, N = 20, the two methods do produce identical results. For other values of N they are not identical but are apparently close.

A.2.2.3 Confidence Limit. When the initial drawing is made to establish 200 sets of 20 shelters each, it was shown in Table A-1 that it would be expected that 71 0-missile sets would be selected if the deployment were legal, i.e., contained 200 missiles. In fact, there would be some variation in this number of 0-missile sets and the confidence in this variation would indicate the degree of acceptance that would be granted the lie upon which the deployer must rely to allow him to use the MCPD technique of Section A.2.3. An estimate of this confidence limit is now made.

The analysis is performed by first determining the distribution of 0-missile, 1-missile, etc. samples by a random selection. It is then shown that this true distribution is approximately normal (for the 0-missile case) so that a conventional

sampling analysis using the normal distribution can be used to produce an estimate of the probability of various numbers of 0-missile sets occurring when the deployment is legal.

From a lot of 4000 shelters containing 200 missiles, a sample of 20 shelters was drawn randomly and the number of missiles found in the sample (=X) observed. This selection was repeated on each of a total of 20 samples (forming a "group"), each sample being chosen from a fresh lot, and the number of samples (=Y) observed in which X-missiles per sample were found in each group. Also observed was the number of groups (=Z) for which X-missiles per sample were found in Y-samples. Ten thousand groups of 20 samples each (for a total of 200,000 samples) were tested.

The technique is described in the following two illustrative tables using three groups and fictitious values for Y and Z.

TABLE A-3. INITIAL RANDOM DATA, EXAMPLE

X	Group Number			Y	Z
	1	2	3		
0	Y = 8	10	8	8	2
				10	1
1	9	5	7	9	1
				5	1
				7	1
2	3	3	3	3	3
3	0	2	0	0	2
				2	1
4	0	0	2	0	2
				2	1
5	0	0	0	0	3
6	0	0	0	0	3
Total Samples	20	20	20		

Table A-3 shows that in the first group of 20 samples X = 0 missiles per sample were found Y = 8 times, X = 1 missiles per sample Y = 9 times, and X = 2 missiles per sample Y = 3 times. These data are then presented in the format of Table A-4.

TABLE A-4. SUMMARY OF RANDOM DATA, EXAMPLE

X	Y: 0	1	2	3	4	5	6	7	8	9	10	Σ Z
0									2		1	3
1						Z = 1		1		1		3
2				3								3
3	2		1									3
4	2		1									3
5	3											3
6	3											3

The results of the random selection are shown in Table A-5 which is in the format of Table A-4 (apparently 10 groups were lost in rounding for the 0-missile and 5-missiles per sample categories).

The distributions of Z on Y in Table A-5 are presumably the truth for the lot. Observing the condition for Z = 0 missiles where Z = 1630 for Y = 8, the probability is 1630/10,000 = 0.163 that 0 missiles will be found in 8 out of 20 samples (= 0.4) for a contribution to the total probability of finding 0-missiles of 0.163 x 0.4 = 0.0652 = YZ/200,000. For the 0-missile case, these contributing probabilities are calculated in Table A-6 and summed to arrive at the hypergeometric solution of 0.3577 probability of finding exactly 0 missiles.

The distribution of the 0-missile probabilities from the second column of Table A-6 is plotted in Fig. A-1 as a function of Y, together with the normal approximation derived from the

TABLE A-5. RESULTS OF RANDOM SELECTION

Z = number of groups that contained X-missiles in Y-samples per group

Y = Number of samples, in one group of 20 samples, that contained X-missiles	X = Number of missiles found in one sample of 20 shelters						
	0	1	2	3	4	5	6
0	0	0	160	2860	7470	9560	9940
1	20	10	680	3730	2220	420	60
2	80	70	1610	2270	290	10	0
3	270	200	2200	850	20	0	
4	650	480	2170	240	0		
5	1220	980	1550	40	0		
6	1700	1490	940	10	0		
7	1830	1770	450	0			
8	1630	1790	170	0			
9	1220	1400	50	0			
10	710	970	20	0			
11	380	510	0				
12	200	210	0				
13	60	80	0				
14	20	30	0				
15	0	10	0				
Total number of groups that contained X-missiles per sample	9990	10,000	10,000	10,000	10,000	9990	10,000

TABLE A-6. PROBABILITY OF FINDING 0-MISSILE SAMPLES IN RANDOM SELECTION

Number of Samples, in One Group of 20 Samples, that Contained 0-Missiles	Number of Groups that Contained 0-Missiles in Y Samples Per Group	Number of Samples that Contained 0-Missiles in Y Samples Per Group	Probability of Finding 0-Missile Samples in Y Samples Per Group
Y	Z	YZ	YZ/200,000
0	0	0	0
1	20	20	0.00010
2	80	160	0.00080
3	270	810	0.00405
4	650	2,600	0.01300
5	1,220	6,100	0.03050
6	1,700	10,200	0.05100
7	1,830	12,810	0.06405
8	1,630	13,040	0.06520
9	1,200	10,980	0.05490
10	710	7,100	0.03550
11	380	4,180	0.02090
12	200	2,400	0.01200
13	60	780	0.00390
14	20	280	0.00140
15	0	0	0
Rounding 7.15	10	72	0.00036
Total	10,000 groups	71,532 samples	0.35766 probability

Probability of finding 0-missile samples
= 71,532/200,000 = 0.3577
(Hypergeometric probability = 0.3577)

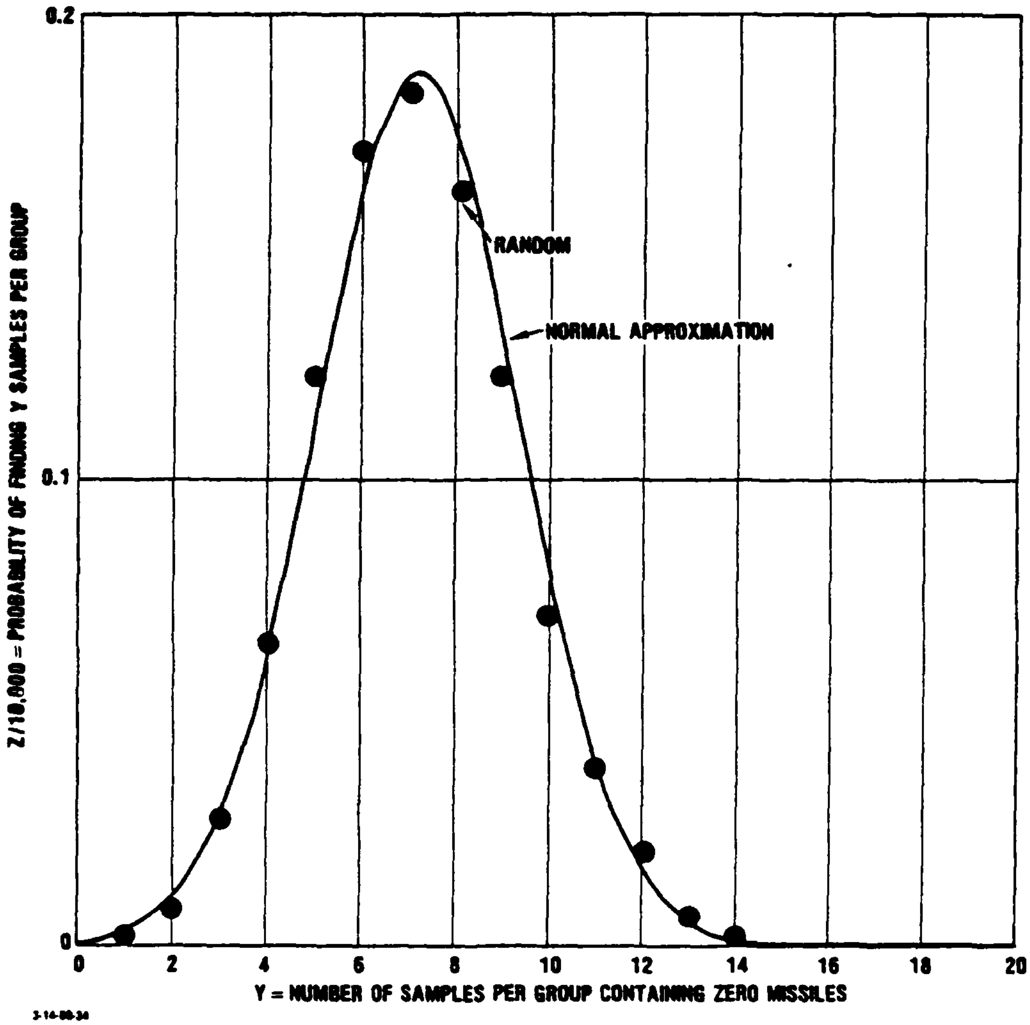

FIGURE A-1. Zero-missile distribution

random distribution, for which $\overline{Y} = 7.154$ and $\sigma = 2.113$. It is believed that the correlation is sufficiently close to justify the use of the normal approximation.

If a sample of size b is taken from a lot of size B, the distribution of the sample is the same as that of the lot, and the standard deviation of the means of the sample (σ') and the mean of the sample standard deviation (d) are:

$$\sigma' = [\sigma/\sqrt{b}]\ [(B - b)(B - 1)]^{1/2} \tag{A-15}$$

$$d = \sigma[(b - 1)/b]^{1/2} \tag{A-16}$$

Where the sample size b = 20 and the lot size B = 4000 as in the analysis, $\sigma' = 0.470$ and $d = 2.100$. If this sample distribution is imposed on the Ys in Table A-6 (rather than the true lot distribution shown there) with means of the sample distribution at various values of $Y = \overline{Y} + t\sigma'$ where $t = (Y - \overline{Y})/\sigma'$ and the calculations of Table A-6 performed, a total probability (when multiplied by 200, the number of 0-missile sets is found) of selecting 0-missiles is obtained for each t. If the distribution is indeed normal, the value of the cumulative normal distribution function from $-\infty$ to t yields the probability of selecting the number of 0-missile sets.

The results are shown in Fig. 2; the hypergeometric solution (71.54 0-missile sets) occurs at 50 percent probability. (Selected values of the probabilities are repeated in the last column of Table A-7.) It appears that there is little opportunity for the deployer to utilize the MCPD declaration unless the number of illegal missiles is small.

A.2.3 Declaration of MCPD Distribution

If the deployer has installed some illegal missiles he could declare a particular distribution of k-missile sets so as to deprive the inspector of the probability of detection (determined in Section A.2.2) obtained by the inspection of only 0-missile sets. In declaring this particular distribution the deployer can force the inspector to consider for inspection both 0-missile and 1-missile sets; this distribution we term the MCPD distribution.

The deployer is constrained to a declaration of the numbers of k-missile sets by the necessity of declaring 200 sets and the legal number (200) of missiles. The declaration used here seems to produce the minimum probability of detection (in 0-missile, 1-missile, and 2-missile sets; the probability of detection is always zero in 3-or-greater-missile sets and usually zero in 2-missile sets) and so is assumed to be the distribution the deployer would declare.

Figure A-2 shows the probability of detection if the inspector chooses either 0-missile or 1-missile sets for the case of M = 120 and various declared distributions. By declaring either 53 0-missile sets and 94 1-missile sets, or 52 0-missile sets and 96 1-missile sets, the deployer can impose a maximum probability of detection of 0.287 or 0.302 on the inspector.

A.2.3.1 Analysis. For any M if the expected number of k-missile sets is G_M (from Table A-2 for k = 0 or 1) and if the declared number of k-missile sets is G_D then the MCPD is

$$P14 = 1 - (G_M/G_D)^{N/20} , \qquad \text{(A-17)}$$

where G_Ds are selected for both k = 0 and k = 1 so that approximately the same probabilities of detection exist (and are therefore the minimum).

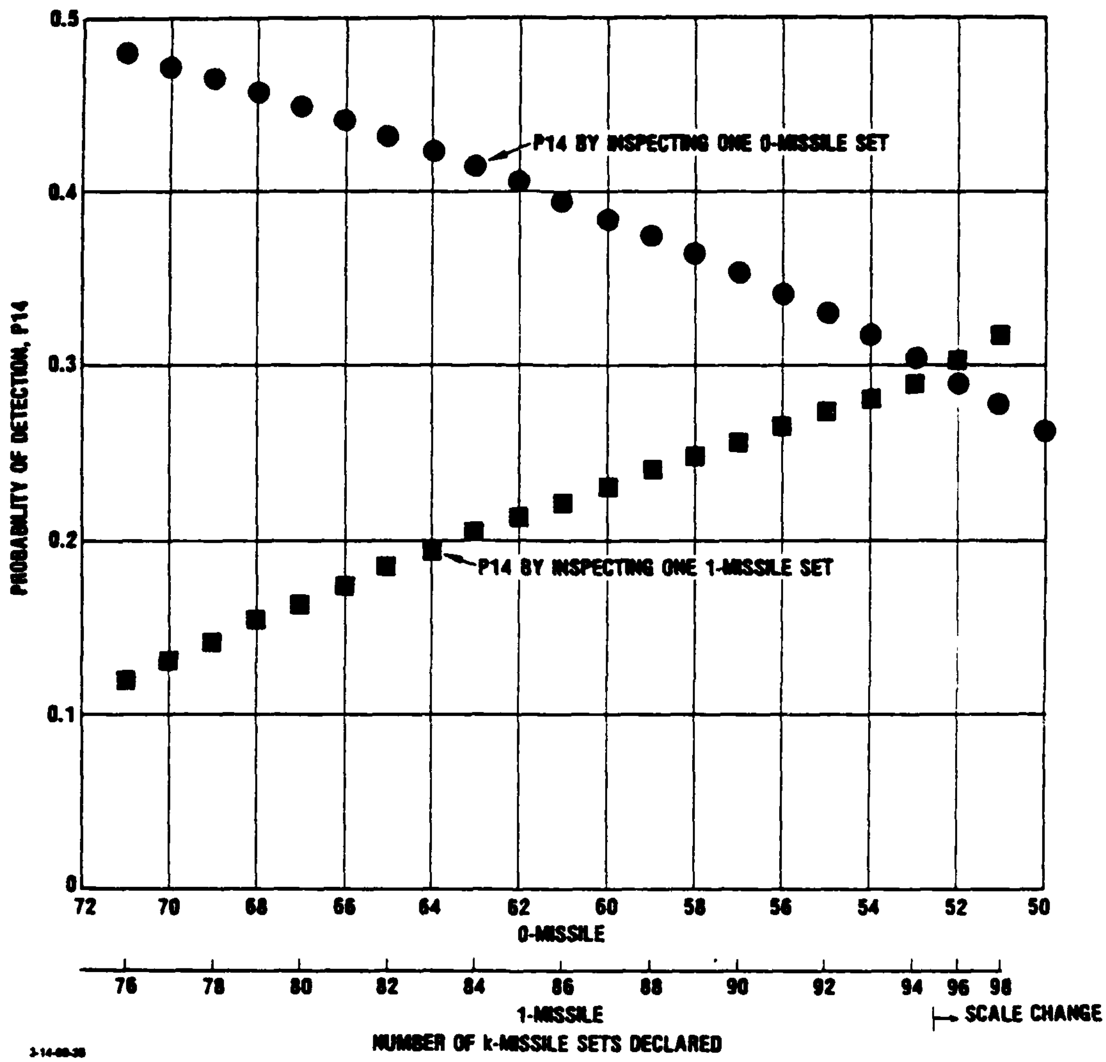

FIGURE A-2. Identification of MCPD distribution for 120 illegal missiles

For M = 20 to 200 the MCPDs (for N = 20) and their associated G_Ds are listed in Table A-7; the MCPDs are plotted in Fig. 4 for N = 20 to 100.

A.4 COOPER'S SAMPLE AND SEARCH

For this paper, the deployment is assumed to consist of 200 (G) groups of 20 shelters each with one legal missile per group.

1. The inspector samples N1 groups by inspecting one shelter in each group. Any group in which a missile is found is a missile-bearing group. (In this analysis, N1 was examined for 5, 10, 15, 20, 40, 60, 80, and 100.)
2. The inspector then is permitted to inspect one or more (A) of the additional shelters in each missile-bearing group; this is called the search phase. (In this analysis, where A is a variable, it was examined for 4, 9, 10, 11, 12, 13, 14, and 19.)

If an illegal number of missiles M were deployed it is assumed that they would be distributed equally among a number of illegal groups (G1). This opportunity is permitted the deployer in the analysis under the assumption that he knows the inspection plan, i.e., N1 and A. The probabilities of detection are determined with the deployer using the best cheating strategy (BCS). The BCS is different for different values of N1 and/or A, and for M, except when N1 = G and A = 19, in which case the BCS expressed as N/G1 is the same for all M.

There are G1 illegal groups containing 20 G1 shelters and M + G1 missiles. The probability, P2, of finding a missile upon sampling an illegal group is

$$P2 = (M + G1)/(20\ G1) \qquad \text{(A-18)}$$

TABLE A-7. MINIMUM COMMON PROBABILITY OF DETECTION DECLARATION

Number of Illegal Missiles	Number of k-Missile Sets Declared			P14 = Probability of Detection in k-Missile Sets, N = 20			Percent Probability of Number of 0-Missile Sets Occurring in Legal Deployment
M	0-Missile	1-Missile	2-Missile	0-Missile	1-Missile	2-Missile	(Figure 2)
20	67	79	41	0.045	0.051	0	24
40	63	81	49	0.079	0.086	0.082	4
60	59	84	55	0.136	0.131	0.109	0.4
80	57	88	53	0.193	0.193	0.038	0.1
100	55	91	53	0.255	0.253	0	0.02
120	52	96	52	0.288	0.302	0	< 0.02
140	52	96	52	0.346	0.344	0	< 0.02
160	49	102	49	0.408	0.412	0	< 0.02
180	49	102	49	0.449	0.441	0	< 0.02
200	47	106	47	0.489	0.491	0	< 0.02

A.4.1 Sample All Groups, Search Missile-Bearing Groups Fully ($N1 = G$, $A = 19$)

A.4.1.1 Probability of Detection. Since every group is sampled, every illegal group is sampled. Upon sampling G1 illegal groups, the probability of detection of a cheat is:

$$P15 = 1 - \left[1 - (M + G1)/(20G1)\right]^{G1} \qquad \text{(A-19)}$$

The best cheating strategy was determined forM = 20, 40, ..., 200 by finding for each M the numbers of illegal groups yielding integers for M/G1 and then the probability of detection, P15. An example is shown in Table A-8 for M = 80; the minimum P15 occurs at M/G1 = 5 and so occurs for every M. The BCS under these inspection rules is 5 illegal missiles per illegal group. With the BCS of M/G1 = 5 (which applies only to this case of N1 = G, A = 19), P2 and P15 become

$$P2 = 0.30 \qquad \text{(A-20)}$$

and

$$P15 = 1 - 0.70^{G1} \quad , \qquad \text{(A-21)}$$

or

$$P15 = 1 - 0.70^{M/5} \quad . \qquad \text{(A-22)}$$

TABLE A-8. EXAMPLE OF DETERMINATION OF BEST CHEATING STRATEGY, M = 80

G1	M + G1	P15	M/G1	
80	160	0.9998	1	
40	120	0.9985	2	
20	100	0.9968	4	
16	96	0.9967	5	Minimum P15, Best Cheating Strategy
10	90	0.9975	8	
8	88	0.9983	10	
5	85	0.9999	16	

A.4.1.2 Expected Number of Shelters Inspected. Two hundred shelters are inspected upon sampling.

There are 200 G1 legal groups and in each legal group the probability of finding a missile upon sampling is 1/20 = 0.05 so that 10 - 0.05G1 legal groups are searched (i.e., A = 19 additional shelters inspected) and 190 - 0.95G1 additional legal shelters are inspected.

There are G1 illegal groups and in each illegal group the probability of finding a missile upon sampling is (M + G1)/(20G1) so that 0.05 M + 0.05G1 illegal groups are searched and 0.95M + 0.95G1 additional illegal shelters are inspected.

The total number of shelters inspected N = 390 + 0.95M. The probability of detection is shown as a function of N in Fig. 5.

The number of shelters inspected is probably larger than that of interest so this scheme is not investigated further--neither is the scheme for sampling all groups (N1 = G) and searching missile-bearing groups partially (A < 19) for the same reason.

A.4.2 Sample Part of Groups, Search Missile-Bearing Groups Fully (N1 < G, A = 19)

A.4.2.1 Probability of Detection. There are two steps; the first is the determination of the probability of selecting an illegal group in the limited-sampling process, and the second (also during sampling) the determination of the probability of finding any missile in an illegal group. Once any missile is found in an illegal group during sampling the cheating is discovered since all additional shelters in the group are searched.

The first step is the selection of N1 groups for sampling out of the total 200 groups (=G) in which there are G1 illegal groups. The hypergeometric distribution states the probability, P5K, of selecting exactly K illegal groups is:

$$P5K = \frac{\binom{G1}{K}\binom{G - G1}{N1 - K}}{\binom{G}{N1}} \tag{A-23}$$

where K is from 0 to G1

The second step, the probability, P2, of finding a missile in an illegal group upon sampling is given in Eq. (A-18). For each K of Eq. (A-23) the probability of not finding a missile upon sampling K illegal groups is

$$P7K = P5K\ (1 - P2)^{K} \ . \tag{A-24}$$

The sum of the P7Ks is thus the total probability of not finding a missile in any illegal group upon sampling. The probability of finding a missile in any illegal group, and thus detecting a cheat since all missile-bearing groups are searched fully, is

$$P4 = 1 - \sum_{K=0}^{K=G1} P7K \quad . \tag{A-25}$$

Table A-9 shows an example to illustrate the process. The example is for M = 40 illegal missiles, G1 = 4 illegal groups, and N1 = 15 groups sampled. The total number of shelters inspected N = 32 (see Section A.4.2.2).

The analysis was performed for M = 20, 40, 80, 120, and 200 illegal missiles, N1 = 5, 10, 15, 20, 40, 60, 80, and 100 groups sampled, and all M/G1s (illegal missiles/illegal group) proving to be integers. For each combination of M, N1, and G1 the probabilities of detection, P4, were computed and the G1 yielding the lowest probability of detection selected as the best cheating strategy. The M/G1 for BCS varies with M, N1, and G1. The results are shown in Fig. 6.

TABLE A-9. EXAMPLE OF DETERMINATION OF PROBABILITY OF DETECTION; $N1 < G$, $A = 19$, $M = 40$, $G1 = 4$, $N1 = 15$

1	2	3	4
K	P5	$(1-P2)^K$	P7
Number of Illegal Groups Selected by sampling	Probability of Selecting Exactly K Illegal Groups in Sampling	Probability of Not Finding a Missile in Sampling K Illegal Groups	Probability of Selecting K Illegal Groups and Not Finding a Missile in Sampling
	Eq. (A-23)	0.450^K	(2) x (3)
0	0.7303	1.0000	0.7303
1	0.2408	0.4500	0.1083
2	0.0276	0.2025	0.0056
3	0.0013	0.0911	0.0001
4	0	0.0410	0
Total	1.0000		0.8443

$P2 = (M + G1)/(20G1) = 0.550$

$1 - P2 = 0.450$

P4 = Probability of Detection = 1 - 0.8443 = 0.1557

A.4.2.2 Expected Number of Shelters Inspected. N1 shelters are inspected upon sampling.

There are 200-G1 legal groups with a probability of sampling each legal group of N1/200 and a probability of finding a missile upon sampling of 1/20 = 0.05 so that 0.05N1 - 0.00025 N1 G1 legal groups are searched (i.e., A = 19 additional shelters are inspected), and 0.95N1 - 0.00475 N1 G1 additional legal shelters are inspected.

There are G1 illegal groups with a probability of sampling each illegal group of N1/200 and a probability of finding a missile upon sampling of (M + G1)/20G1 so that 0.00025 N1 M + 0.00025 N1 G1 illegal groups are searched and 0.00475 N1 M + 0.00475 N1 G1 additional illegal shelters are inspected.

The total number of shelters inspected N = 1.95N1 + 0.00475 N1 M. The probability of detection is shown as a function of N in Fig. 6.

A.4.3 Sample Part of Groups, Search Missile-Bearing Groups Partially (N1 < G, A < 19)

A.4.3.1 Probability of Detection. There are again two steps as in Section A.4.2.1 where all of the additional shelters (A = 19) in any missile-bearing group are searched. The first step is the same, i.e., Eq. (A-23) applies for the determination of P5K, the probability of selecting exactly K illegal groups upon sampling.

The second step differs from that in Section A.4.2.1. When an illegal group's additional shelters, numbering A, are searched, the probability, P3, of finding an illegal missile is 1 minus the probability of not finding an illegal missile. Again, the hypergeometric distribution is called for, producing

$$P3 = 1 - \frac{\binom{M/G1}{K}\binom{19 - M/G1}{A - K}}{\binom{19}{A}} , \qquad \text{(A-26)}$$

where

19 = the total number of shelters in a group available for search

M/G1 = the number of illegal missiles per illegal group

A = the number of additional shelters per group searched

K = 0 = the number (none) of illegal missiles to be found

The probability, P2, of finding a missile in an illegal group upon sampling is (same as in Section A.4.2.1) given in Eq. (A-18). The probability, P8, that a missile is found in an illegal group upon sampling and that an illegal missile is then found when A additional shelters of the illegal group are searched is

$$P8 = (P3)\ (P2) \cdot \qquad \text{(A-27)}$$

For each K of Eq. (A-23) the probability of not finding an illegal missile upon sampling and searching A additional shelters of K illegal groups is

$$P9K = P5K\ (1 - P8)^{K} . \qquad \text{(A-28)}$$

The sum of the P9Ks is thus the total probability of not finding an illegal missile upon sampling and searching A additional shelters in all of the illegal groups. The probability of finding an illegal missile in all of the illegal groups is

$$P1 = 1 - \sum_{K=0}^{K=G1} P9K . \qquad \text{(A-29)}$$

Table A-10 shows an example to illustrate the process. The example is for M = 40 illegal missiles, G1 = 4 illegal groups, N1 = 15 groups sampled (thus for the same as Table A-9 where A = 19), and A = 4 additional shelters searched in each

TABLE A-10. EXAMPLE OF DETERMINATION OF PROBABILITY OF DETECTION: N1 < G, A < 19, M = 40, G1 = 4, N1 = 15, A = 4

1	2	3	4
K	$P5$	$(1-P8)^K$	$P9$
Number of Illegal Groups Selected by Sampling	Probability of Selecting Exactly K Illegal Groups in Sampling	Probability of Not Finding an Illegal Missile in Sampling and Searching K Illegal Groups	Probability of Selecting K Illegal Groups and Not Finding an Illegal Missile in Sampling and Searching K Illegal Groups
	Eq. (A-23)	0.4679^K	(2) x (3)
0	0.7303	1.0000	0.7303
1	0.2408	0.4679	0.1126
2	0.0276	0.2189	0.0060
3	0.0013	0.1024	0.0001
4	0	0.0479	0
Total	1.0000		0.8490

P2 = (M + G1)/(20G1) = 0.550

P3 = Eq. (A-26) = 0.9675

P8 = (P3)(P2) = 0.5321

1 - P8 = 0.4679

P1 = Probability of Detection = 1 - 0.8490 = 0.1510

illegal group. The total number of shelters inspected N = 19 (see Section A.4.3.2). The probability of detection, P1, for A = 4 is 0.1510, which is less than the probability of detection, P4 for A = 19 of 0.1557 but it occurs at a lower number of shelters inspected, at N = 19 for A = 4, while at N = 32 for A = 19. Thus, the A and N1 for the best probability of detection must be determined for any M and N by comparing the detection probabilities for all A and N1 combinations, giving the deployer the best cheating strategy for the declared inspection strategy, A and N1.

The comparison was made for N1 = 5, 10, 15, 20, 40, 60, 80, and 100 groups sampled, and A = 4, 9, 10, 11, 12, 13, 14, and 19 additional shelters searched in each missile-bearing group. It was determined that A = 11 is the best inspection strategy and a comparison of the probabilities of detection is shown in Fig. A-3 for 20 and 100 shelters inspected and A = 4, 11, and 19. The best cheating strategy is noted in Fig. A-3 and for A = 11 it is one illegal missile per illegal group when 100 shelters are inspected but is from 10 to 16 (the most dense illegal missiles/group available under the integer rule) when 20 shelters are inspected. The numbers of groups sampled are also shown in Fig. A-3; they vary from low figures when the cheating fraction is high to somewhat higher figures when the cheating fraction is low. The numbers of groups sampled are large when more shelters are inspected but they increase only in proportion to the numbers of shelters inspected.

The results are shown for A = 11, the best inspection strategy, in Fig. 7.

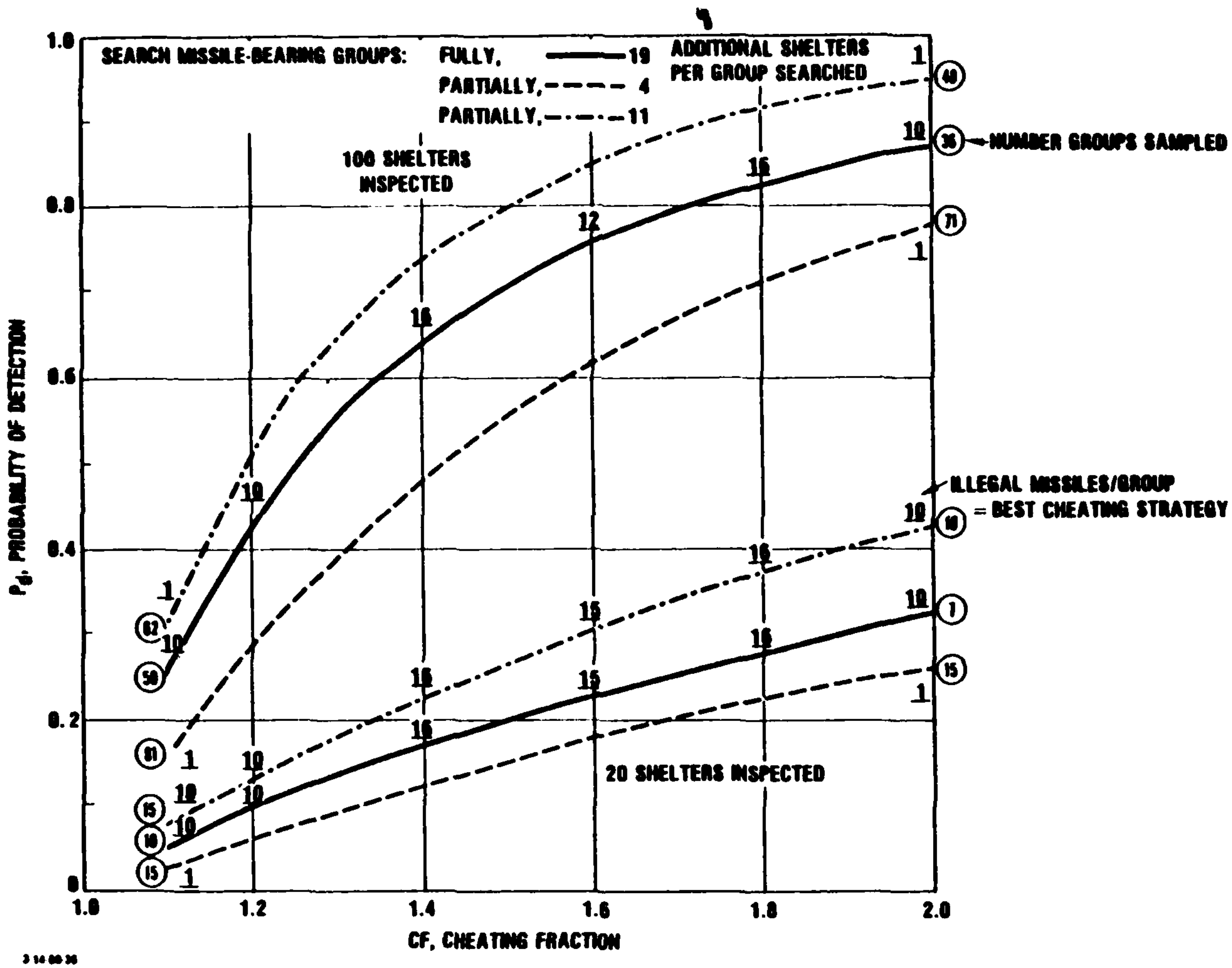

FIGURE A-3. Cooper's sample and search. Sample part of groups, search missile-bearing groups partially.

REFERENCE, APPENDIX

A-1. Feller, William, "An Introduction to Probability Theory and its Applications," Vol. I, Second Edition, John Wiley and Sons, Inc., 1957.

PRECEDING PAGE BLANK-NOT FILMED

NOMENCLATURE

A	number of additional shelters searched in each group, Cooper
b	size of sample (=20)
B	size of lot (=4000)
BCS	best cheating strategy, Cooper
CF	cheating fraction, CF = 1 + M/200 where 200 is the legal number of missiles
d	normal distribution mean of the sample deviation
G	total number of groups, 1 group = 20 shelters, Cooper
G_D	declared number of k-missile sets, Schultis 2 MCPD
G_M	expected number of k-missile sets for M illegal missiles, Schultis 2
G_o	expected number of k-missile sets in a legal deployment, Schultis 2
G1	number of groups containing illegal missiles, Cooper
k	number of missiles in a set of 20 shelters, Schultis 2
K	number of illegal groups selected for sampling, sampling part of groups, Cooper
K'	number of missile-bearing shelters to be found, TRW Method II
m	total number of missiles in a lot of s shelters, Schultis 2
M	total number of illegal missiles deployed
MCPD	minimum common probability of detection, Schultis 2
N	number of shelters inspected
N1	number of groups sampled, Cooper
P1	probability of finding an illegal missile in all of the illegal groups upon sampling part of groups and searching missile-bearing groups partially, Cooper
P2	probability of finding a missile upon sampling an illegal group, Cooper

PRECEDING PAGE BLANK-NOT FILMED

DISTRIBUTION LIST FOR IDA PAPER P-1478

DEPARTMENT OF DEFENSE

Deputy Under Secretary of Defense for Policy Planning
Director DoD SALT Task Force
Room 5A670, The Pentagon
Washington, D.C. 20301
Attn: Dr. L. Davis
Mr. L. Minichiello
Dr. G. Schneiter

Deputy Under Secretary of Defense, Research and Engineering (Strategic and Space Systems)
Director, Offensive and Space Systems
Room 3E129, The Pentagon
Washington, D.C. 20301
Attn: Dr. M. Atkins

Assistant to the Secretary (Atomic Energy)
Director, SALT/Arms Control Support Group
Room 3C125, The Pentagon
Washington, D.C. 20301
Attn: Dr. J. Wade
Mr. G. Butler

Joint Chiefs of Staff
Director (Plans and Policy) J-5
Assistant Deputy Director for International Negotiations
Room 2E1014, The Pentagon
Washington, D.C. 20301
Attn: BG J. Skaff

Director (Plans and Policy) J-5
Strategic Negotiations Division
Room 2D990, The Pentagon
Washington, D.C. 20301
Attn: Col F. Jenkins

Defense Intelligence Agency
Deputy Director for Defense Intelligence
International Negotiations Support
Washington, D.C. 20301
Attn: COL J. Farrar, Jr., DI5

Department of Defense, Continued

Assistant Vice Director for Scientific and Technical Intelligence
Weapons and Systems Branch
Ballistic Missile Systems Branch
Washington, D.C. 20301
Attn: Mr. J. Miller, DT2A

National Security Agency
Fort George C. Meade, MD 20755
Attn: Col R. Leverette

Defense Technical Information Center (12)
Cameron Station
Alexandria, VA 22314

DEPARTMENT OF THE ARMY

Assistant Secretary of the Army (Research, Development, and Acquisition)
Room 2E672, The Pentagon
Washington, D.C. 20310
Attn: Dr. P. Pierre

DEPARTMENT OF THE NAVY

Assistant Secretary of the Navy (Research, Engineering, and Systems)
Room 4E376, The Pentagon
Washington, D.C. 20350
Attn: Dr. D. Mann

DEPARTMENT OF THE AIR FORCE

Assistant Secretary of the Air Force (Research, Development, and Logistics)
Deputy for Strategic and Space Systems
Room 4D977, The Pentagon
Washington, D.C. 20330
Attn: Dr. H. Cooper, SAFALR

Assistant Chief of Staff, Studies and Analysis, USAF
Room 1E388, The Pentagon
Washington, D.C. 20330
Attn: Maj M. Best, AF/SASM

Scientific Advisory Board, USAF
Room 5D982, The Pentagon
Washington, D.C. 20330

Department of the Air Force, Continued

Deputy Chief of Staff, Operations, Plans, and Readiness
Directorate of Plans
Strategic Offensive Forces Division
Room 4D1018, The Pentagon
Washington, D.C. 20330
Attn: Maj T. Kuenning, Jr., AFXOX

Deputy Chief of Staff, Research, Development, and Acquisition, USAF
Directorate of Operational Requirements
Special Assistant for MX
Room 5E373, The Pentagon
Washington, D.C. 20330
Attn: Lt Col K. Van Dillen, AF/RDQF

U.S. Air Force Ballistic Missile Office/MN-1
Norton Air Force Base, CA 92409
Attn: Col A. Casey

U.S. GOVERNMENT

National Security Council
Room 383, Old Executive Office Building
Washington, D.C. 20506
Attn: Mr. F. Ermarth
Dr. R. Molander
Maj Gen J. Welch, Jr.

Department of State
Bureau of Political Military Affairs
Washington, D.C. 20590
Attn: Mr. R. Bartholomew
Dr. C. Henkin

U.S. Arms Control and Disarmament Agency
Department of State
Washington, D.C. 20590
Attn: Dr. A. Lieberman
Mr. J. Timbie

Central Intelligence Agency
Washington, D.C. 20505
Attn: Mr. R. Huffstutler
Mr. R. McCrory

CONTRACTORS

Analytical Assessment Corporation
P.O. Box 9758
Marine del Rey, CA 90291
Attn: Mr. F. Leykham

The Boeing Company
P.O. Box 3707
Seattle, WA 98124
Attn: Mr. J. Blaylock, 85-45
Mr. J. Russel, 85-20

Hudson Institute, Incorporated
Quaker Ridge Road
Croton-on-Hudson, NY 10520
Attn: Mr. C. Gray

Martin Marietta Aerospace
P.O. Box 179
Denver, CO 80201
Attn: Mr. J. Springer

McDonnell-Douglas Corporation
5301 Bolsa Avenue
Huntington Beach, CA 92647
Attn: Mr. R. Nuttall

The Rand Corporation
1700 Main Street
Santa Monica, CA 90406
Attn: Dr. I. Blumenthal

Harold Rosenbaum Associates, Incorporated
40 Mall Road
Burlington, MA 01803
Attn: Dr. H. Rosenbaum

Science Applications Incorporated
8300 Westpark Drive
McLean, VA 22102
Attn: Dr. H. Sauerwein

System Planning Corporation
1500 Wilson Boulevard
Arlington, VA 22209
Attn: Mr. S. Graybeal

Contractors, Continued

TRW Systems Group
San Bernardino Operations
P.O. Box 1310
San Bernardino, CA 92402
Attn: Mr. J.M. Gorman

Tetra Tech, Incorporated
1911 North Fort Meyer Drive
Arlington, VA 22209
Attn: Mr. I. Rutberg

Institute for Defense Analyses

R.H. Fox - STD
W.J. Schultis - ISAD
V.J. Berinati - STD
P. Gould - SED
J.H. Henry - ISAD